Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 1

24 October 2024

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

217

A004

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

10 pages

8824–7109

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Consider the following set of ordered data.

22	22	26	27	28	32	36	38	39	42

(a) Write down

(i) the mode;

(ii) the range;

(iii) the median. [3]

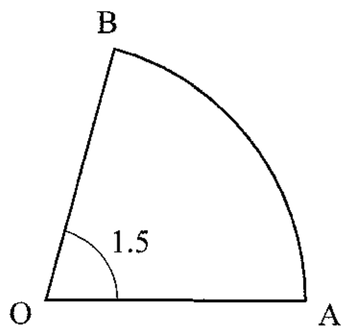
(b) Find the interquartile range. [3]

2. [Maximum mark: 5]

Points A and B lie on a circle with centre O and radius r cm, where $A\hat{O}B = 1.5$ radians.

This is shown on the following diagram.

diagram not to scale



The area of sector OAB is 48 cm^2.

(a) Find the value of r. [3]

(b) Hence, find the perimeter of sector OAB. [2]

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3. [Maximum mark: 6]

Two events A and B are such that $P(A) = 0.65$, $P(B) = 0.45$ and $P(A \cup B) = 0.85$.

(a) Find $P(A \cap B)$. [3]

(b) Find $P(A' \mid B')$. [3]

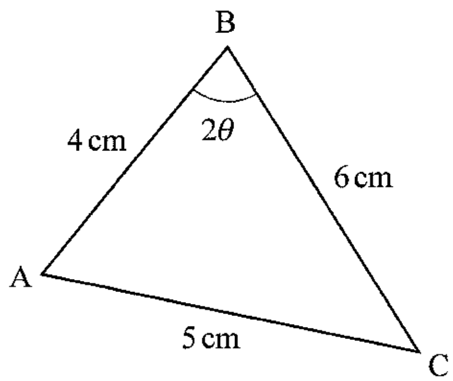
4. [Maximum mark: 4]

Prove that $(3n+2)^2 - (3n-2)^2$ is a multiple of 12 for all $n \in \mathbb{Z}^+$.

5. [Maximum mark: 6]

The following diagram shows triangle ABC, where $AB = 4\,\text{cm}$, $BC = 6\,\text{cm}$, $AC = 5\,\text{cm}$ and $A\hat{B}C = 2\theta$.

diagram not to scale



Find the exact value of $\cos\theta$, giving your answer in the form $\frac{p\sqrt{2}}{q}$, where $p, q \in \mathbb{Z}^+$.

6. [Maximum mark: 6]

For a particular arithmetic sequence, $u_{10} = 16$ and $S_{25} = 100$.

Find the value of k such that $u_k = 0$.

Do **not** write solutions on this page.

Section B

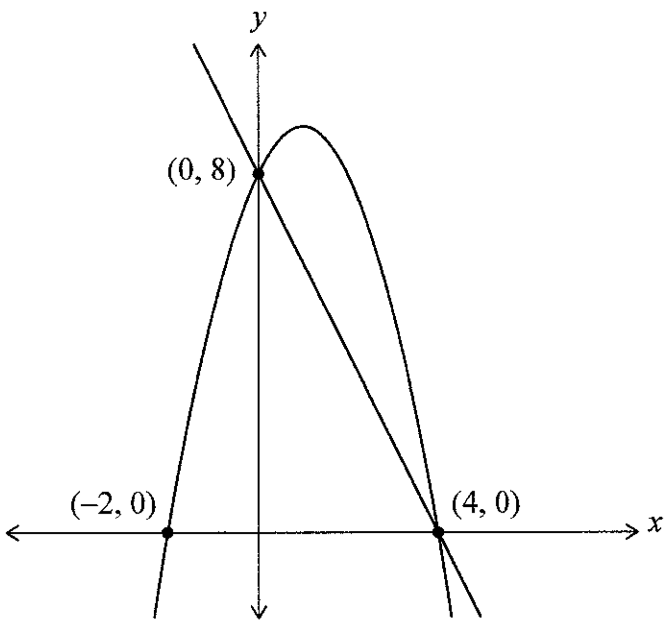
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

The following diagram shows parts of the graphs of two functions f and g.

The graph of f is linear, has an x-intercept at $(4, 0)$ and a y-intercept at $(0, 8)$.

The graph of g has x-intercepts at $(-2, 0)$ and $(4, 0)$ and a y-intercept at $(0, 8)$.



(a) Write down the equation for f in the form $f(x) = mx + c$. [2]

The function g is given by $g(x) = -x^2 + bx + 8$, where b is a real constant.

(b) Find the value of b. [3]

(c) Show that the area of the region enclosed by the graph of f and the graph of g can be represented by the definite integral $\int_0^4 (-x^2 + 4x)\mathrm{d}x$. [2]

(d) Hence, find the area of the region enclosed by the graph of f and the graph of g. [4]

Point P is on the graph of g. The tangent to the graph of g at P is parallel to the graph of f.

(e) Find the coordinates of P. [5]

Do **not** write solutions on this page.

8. [Maximum mark: 14]

The function f is defined as $f(x) = \log_2(4x)$, where $x > 0$.

(a) Find the value of

(i) $f(8)$;

(ii) $f\left(\frac{1}{4}\right)$. [3]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

The graph of $y = f(16x^3)$ can be obtained by translating and stretching the graph of $y = \log_2 x$.

(d) Describe these two transformations specifying the order in which they are to be applied. [6]

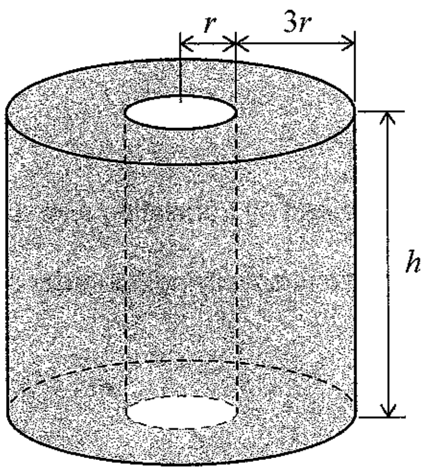
Do **not** write solutions on this page.

9. [Maximum mark: 17]

Consider a cylinder of radius $4r$ and height h. A smaller cylinder of radius r is removed from the centre to form a hollow cylinder. This is shown in the following diagram.

All lengths are measured in centimetres.

diagram not to scale



The total surface area of the hollow cylinder, in cm^2, is given by S.

The volume of the hollow cylinder, in cm^3, is given by V.

(a) Show that $S = 30\pi r^2 + 10\pi rh$. [3]

(b) The total surface area of the hollow cylinder is $240\pi\,\text{cm}^2$.

Show that $V = 360\pi r - 45\pi r^3$. [6]

(c) Find an expression for $\frac{\mathrm{d}V}{\mathrm{d}r}$. [2]

The hollow cylinder has its maximum volume when $r = p\sqrt{\frac{2}{3}}$, where $p \in \mathbb{Z}^+$.

(d) Find the value of p. [3]

(e) Hence, find this maximum volume, giving your answer in the form $q\pi\sqrt{\frac{2}{3}}$, where $q \in \mathbb{Z}^+$. [3]